Saginaw Valley State University

2007 Math Olympics – Level I

1. A middle school has 100 lockers numbered 1 to 100, and 100 students. The first student goes down the row of lockers and opens every locker. Then the second student goes down the row of lockers and closes every locker that is numbered with a multiple of two. Then the third student goes down the row of lockers, and for every locker that is numbered with a multiple of 3, if it is open, she closes it, but if it is already closed, she opens it again. The fourth student then does the same thing for the lockers numbered with multiple of 4, and so on, down to the hundredth student. In the end, how many lockers are still open?

   (a) 1 **(b)** all of the lockers that are not numbered with prime numbers **(c)** 10

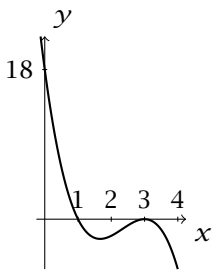
   (d) 15 **(e)** None of the above

2. Which of the following equations describes the set of all points that are equidistant from the points $P(-1,3)$ and $Q(3,5)$?

   (a) $x-2y=-7$ **(b)** $(x-1)^2+(y-4)^2=5$ **(c)** $(x+1)^2+(x-3)^2=5$

   (d) $2x+y=6$ **(e)** None of the above

3. Which of the following expressions is a factored form of the third degree polynomial function $f(x)$ whose graph is given?



   (a) $18(x-1)(x-3)^2$ **(b)** $-2(x-1)(x-3)^2$

   (c) $18(x-1)^2(x-3)$ **(d)** $-6(x-1)^2(x-3)$ **(e)** None of the above

4. Find the length of a chord that is 5 cm from the center of a circle with radius 13 cm.

   (a) 12 cm **(b)** 24 cm

   (c) 13 cm **(d)** Not enough information given

   (e) None of the above

5. Find the area of a triangle (in square units) bounded by the coordinate axes and the line $x+3y-12=0$.

   (a) 12 **(b)** 18 **(c)** 24 **(d)** 48 **(e)** None of the above

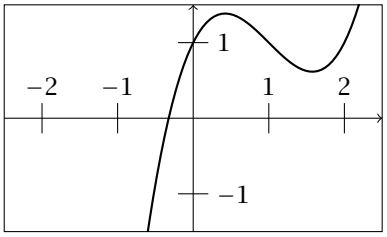
6. Augustus, Benedict, Claudio, and Diana have been accused of stealing the golden mean. It is known that one of these four people must have done it. Augustus says "Benedict did it". Benedict says "Diana did it". Claudio says "I didn't do it". Diana says "Benedict is lying when he says I did it". If it is known that exactly one of them is lying, which one did it?

   (a) Augustus **(b)** Benedict

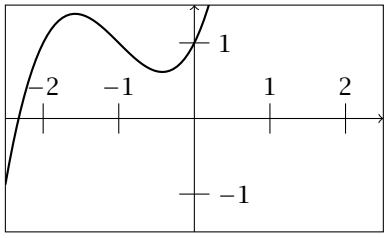
   (c) Claudio **(d)** Diana

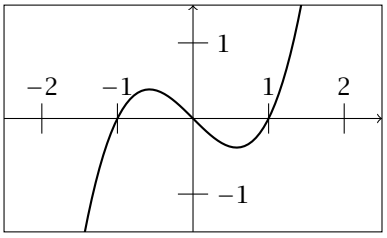
   (e) More than one person must be lying

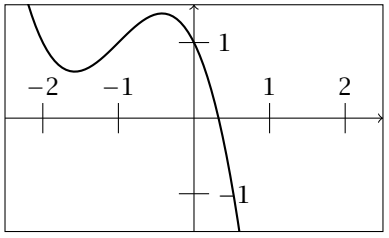
7. The function f has graph

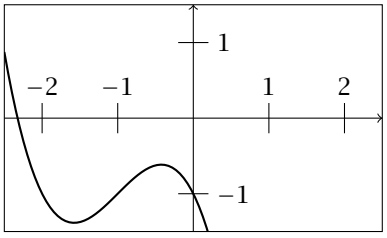


   Which of the following could be the graph of $f(x+1)-1$?

(a)


(b)


(c)


(d)


   (e) None of the above

8. The sum of the real solutions to the equation $(x^3-1)^3 - 7(x^3-1)^2 - 4(x^3-1) + 28 = 0$ is

   (a) 7 **(b)** $\sqrt[3]{7}$ **(c)** $2+\sqrt[3]{3}$ **(d)** $1+\sqrt[3]{3}$ **(e)** None of the above

9. $\left(\frac{5}{4}-1\right)^{-\frac{1}{2}}$ is equal to

   (a) 2 **(b)** $-\frac{1}{\sqrt{5}}$ **(c)** $\frac{1}{25}$ **(d)** $\frac{1}{16}$ **(e)** None of the above

10. You can paint your living room in 6 hours and your friend would take 8 hours to do the same job. How long will it take the two of you to paint the living room if you work together?

(a) 21 minutes **(b)** $3\frac{3}{7}$ hours **(c)** 3 hours and 24 minutes

(d) $3\frac{1}{2}$ hours **(e)** None of the above

11. Assume X, Y and Z represent positive real numbers. The expression $\left(\frac{16X^{-6}Y^{8}}{Z^{\frac{4}{3}}}\right)^{-\frac{3}{4}}$ is equivalent to

(a) $\frac{8X^{\frac{9}{2}}Z}{Y^{6}}$ **(b)** $\frac{ZX^{4}\sqrt{X}}{8Y^{6}}$ **(c)** $\frac{Y^{\frac{32}{3}}}{16X^{8}Z^{\frac{16}{9}}}$ **(d)** $-\frac{8X^{\frac{9}{2}}Y^{6}}{Z}$ **(e)** None of the above

12. 10% of a high school senior class participate in Math Olympics. 95% of the seniors that participate in Math Olympics get into the college of their choice. Only 50% of the seniors who don't participate in Math Olympics get into the college of their choice. What percentage of seniors from that high school get into the college of their choice?

(a) 9.5% **(b)** 54.5% **(c)** 59.5% **(d)** 60% **(e)** None of the above

13. How many two digit numbers are such that when the tens digit and the ones digit are interchanged, the resulting two digit number is 9 more than the original two digit number? (Note that 0 cannot be the first digit of a two digit number.)

(a) 0 **(b)** 1 **(c)** 8 **(d)** 9 **(e)** None of the above

14. Which of the following is an equation of a circle with diameter that has endpoints $P(2,5)$ and $Q(6,-3)$.

(a) $(x-4)^2+(y-1)^2=20$ **(b)** $(x-4)^2+(y-1)^2=40$ **(c)** $(x-2)^2+(y-4)^2=20$

(d) $(x-2)^2+(y-4)^2=40$ **(e)** None of the above

15. Rationalize the denominator: $\frac{4\sqrt{3}}{2+\sqrt{3}+\sqrt{7}}$

(a) $\frac{4}{7}$ **(b)** $2+\sqrt{3}-\sqrt{7}$ **(c)** $-\frac{4\,(2-\sqrt{7})}{3}$ **(d)** $2\sqrt{3}+4+\frac{4\sqrt{21}}{7}$

(e) None of the above

16. Find the value(s) of the parameter b, if possible, such that the linear system

$$\begin{cases} x + 2y = 1 \\ 3x + by = 3 \end{cases}$$

has infinitely many solutions.

(a) $b = -2$ only

(b) $b = 6$ only

(c) Both $b = -2$ and $b = 6$

(d) There are infinitely many such choices of b

(e) There is no such choice of b

17. To color the interior of a square of side r ft, we require exactly one can of paint. We also know we need exactly one can of paint to color the interior of a right triangle whose legs measure $(r + \frac{3}{2})$ ft and 4 ft. Assuming that the area that can be colored depends only on the amount of paint (and no other factors), what is the minimal number of cans of paint we need to buy to paint the interior of a 6 ft by 9 ft rectangle?

(a) 6 cans **(b)** 7 cans **(c)** 8 cans **(d)** 9 cans **(e)** None of the above

18. Suppose that $f(x) = 2\,(f(x+1) + f(x-1))$ for all x. If $f(2) = 2$ and $f(4) = -2$, what is $f(7)$?

(a) -1 **(b)** $\frac{3}{2}$ **(c)** $\frac{7}{4}$ **(d)** $-\frac{5}{8}$ **(e)** None of the above

19. The radius of a circle inscribed in an equilateral triangle is $2\sqrt{3}$ in. Find the exact area of the triangle.

(a) 12π in^2 **(b)** 36 in^2 **(c)** $36\sqrt{3}$ in^2 **(d)** $18\sqrt{3}$ in^2 **(e)** None of the above

20. The lengths of the three sides of a triangle are 13 cm, 14 cm and 15 cm. Find the length of the altitude to the 14 cm side.

(a) 5 cm

(b) 9 cm

(c) 12 cm

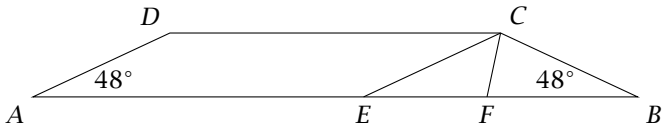
(d) Not enough information given

(e) None of the above

21. A farmer needs to shorten the width of a bean field by 20% to make room for a storage shed. He would like the bean field to keep the same area. By what percent should he increase the length of the field?

(a) 15% **(b)** 20% **(c)** 24% **(d)** 25% **(e)** None of the above

22. Given: $\overline{AB} \parallel \overline{DC}$, $\overline{CE} \parallel \overline{DA}$
$m\angle A = m\angle B = 48°$
$\overline{CF}$ bisects $\angle DCB$
Find: $m\angle FCE$



(a) 18° **(b)** 33°

(c) 42° **(d)** Not enough information given

(e) None of the above

23. Let F be a function such that $F\left(\frac{8}{\sqrt{1+\sqrt{x}}}\right) = x$ for all $x \geq 0$. What is $F(4)$?

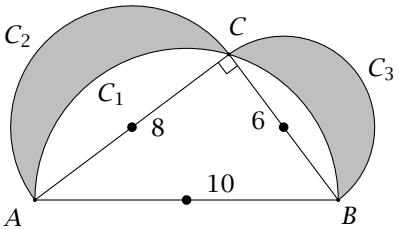
(a) 9 **(b)** $\frac{8\sqrt{3}}{3}$ **(c)** $\frac{-12+8\sqrt{3}}{3}$ **(d)** $\frac{9}{16}$ **(e)** None of the above

24. The solution to the inequality $\dfrac{x-1}{x-2} \geq 3$ is

(a) $x \leq \frac{5}{2}, x \neq 2$ **(b)** $x < 2$ or $x \geq \frac{5}{2}$ **(c)** $2 < x \leq \frac{5}{2}$ **(d)** $2 \leq x \leq \frac{5}{2}$

(e) None of the above

25. A right triangle with sides 6, 8 and 10 is inscribed in a circle C_1. The legs of the right triangle are the diameters of the half circles that lie outside the triangle as shown. Find the area of the shaded region.



(a) 28

(b) 28π **(c)** 24 **(d)** 24π **(e)** None of the above